Important Questions

Term-02 Exam (2022)

Sub - Maths

Class - XII

Edit 27/03/22
S. SHARMA
PGT (Maths)
GHSS, Naharlagun
8729900980

Q No.

1. $\int \frac{5x+3}{\sqrt{x^2+4x+10}} \cdot dx$

Parts : $L = A \frac{d}{dx}(Q) + B$

$$5x+3 = A\frac{d}{dx}(x^2+4x+10) + B$$

$$5x+3 = A(2x+4) + B$$

Coeff of x, $\quad 5 = 2A \quad \therefore A = 5/2$

Const., $\quad 3 = 4A + B$

$$3 = \cancel{4}^{2}\left(\frac{5}{\cancel{2}}\right) + B \quad \therefore B = -7$$

Integrals : $\int \frac{A(2x+4)}{\sqrt{x^2+4x+10}} + \frac{B}{\sqrt{x^2+4x+10}} \cdot dx$

$$= \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}} \cdot dx - 7\int \frac{1}{\sqrt{x^2+4x+4-4+10}} \cdot dx$$

$$= \frac{5}{\cancel{2}} \cdot \cancel{2}\sqrt{x^2+4x+10} - 7\int \frac{1}{\sqrt{(x+2)^2+(\sqrt{6})^2}} \cdot dx$$

$$= 5\sqrt{x^2+4x+10} - 7\log\left|(x+2) + \sqrt{x^2+4x+10}\right| + C$$

Q.No:

2. $$\int \frac{x^2}{(x^2+1)(x^2+4)} \cdot dx$$

let $x^2 = P$ for P.F.

$$= \int \frac{P}{(P+1)(P+4)} \cdot dx$$

Partial Fraction

$$\frac{P}{(P+1)(P+4)} = \frac{A}{(P+1)} + \frac{B}{(P+4)}$$

$$\frac{P}{\cancel{(P+1)(P+4)}} = \frac{A(P+4) + B(P+1)}{\cancel{(P+1)(P+4)}}$$

$$P = A(P+4) + B(P+1)$$

Put $P=-1$, $\quad -1 = A(-1+4) \quad \therefore A = -1/3$

$P=-4$, $\quad -4 = B(-4+1) \quad \therefore B = 4/3$

Integrals $\int \frac{A}{P+1} + \frac{B}{P+4} \cdot dx$

$$= -\frac{1}{3}\int \frac{1}{x^2+1}\, dx + \frac{4}{3}\int \frac{1}{x^2+2^2} \cdot dx$$

$$= -\frac{1}{3}\tan^{-1}x + \frac{\overset{2}{\cancel{4}}}{3}\cdot\frac{1}{\cancel{2}}\tan^{-1}\frac{x}{2} + C$$

$$= -\frac{1}{3}\tan^{-1}x + \frac{2}{3}\tan^{-1}\frac{x}{2} + C$$

Q.No

3. $= \int \frac{x \cos^{-1}x}{\sqrt{1-x^2}} \cdot dx$

$$= \int \cos^{-1}x \cdot \frac{x}{\sqrt{1-x^2}} \cdot dx$$

$$= \frac{1}{-2} \int \underbrace{\cos^{-1}x}_{\mathrm{I}} \cdot \underbrace{\frac{-2x}{\sqrt{1-x^2}}}_{\mathrm{II}} \cdot dx, \qquad \text{ILATE}$$

$$= -\frac{1}{2}\left[\cos^{-1}x \cdot \int \frac{-2x}{\sqrt{1-x^2}} dx - \int\left(\frac{d}{dx}(\cos^{-1}x) \int \frac{-2x}{\sqrt{1-x^2}} dx\right) \cdot dx\right]$$

$$= -\frac{1}{2}\left[\cos^{-1}x \cdot 2\sqrt{1-x^2} - \int \frac{-1}{\cancel{\sqrt{1-x^2}}} \cdot 2\cancel{\sqrt{1-x^2}} \cdot dx\right]$$

$$= -\frac{1}{\cancel{2}}\left[\cancel{2}\sqrt{1-x^2}\cos^{-1}x + \cancel{2}\int 1 \cdot dx\right]$$

$$= -\left[\sqrt{1-x^2}\cos^{-1}x + x\right] + C$$

Q.No (4.)

$$\int_2^8 |x-5| \cdot dx$$

using property of DI

$$= \int_2^5 -(x-5)\, dx + \int_5^8 (x-5) \cdot dx$$

$$= -\left[\frac{x^2}{2} - 5x\right]_2^5 + \left[\frac{x^2}{2} - 5x\right]_5^8$$

$$= -\left[\left(\frac{25}{2} - 25\right) - (2-10)\right] + \left[(32-40) - \left(\frac{25}{2} - 25\right)\right]$$

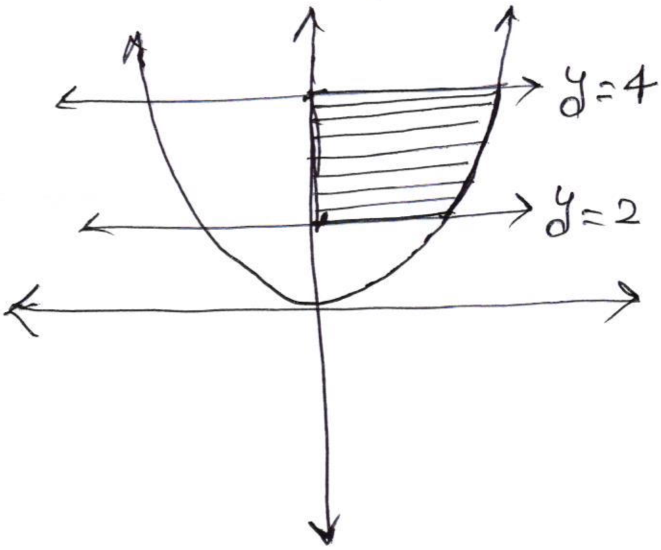
$$= -\left[\frac{-25}{2} + 8\right] + \left[-8 + \frac{25}{2}\right]$$

$$= \frac{9}{2} + \frac{9}{2}$$

$$= 9$$

Q.No (5.) Find the area bounded by $x^2 = 4y$, $y=2$, $y=4$ and y-axis in 1^{st} quadrant.

Sol:



area using integration

$$= \int_2^4 2\sqrt{y} \cdot dy \qquad \left\{ \begin{array}{l} x^2 = 4y \\ x = 2\sqrt{y} \end{array} \right.$$

$$= 2 \cdot \left[\frac{y^{3/2}}{3/2} \right]_2^4$$

$$= \frac{4}{3} \left[y^{3/2} \right]_2^4$$

$$= \frac{4}{3} \left[4^{3/2} - 2^{3/2} \right]$$

$$= \frac{4}{3} \left[8 - 2\sqrt{2} \right]$$

$$= \frac{8}{3} \left[4 - \sqrt{2} \right] \text{ sq. units}$$

Q.No

6. Solve the DE

$$e^x \tan y \, dx + (1-e^x) \sec^2 y \, dy = 0$$

Sol: $$e^x \tan y \, dx + (1-e^x) \sec^2 y \, dy = 0$$

$$(1-e^x) \sec^2 y \, dy = -e^x \tan y \, dx$$

$$\frac{\sec^2 y}{\tan y} dy = -\frac{e^x}{1-e^x} dx$$

it is vareiable separable Diff. Eq. (VSDE)

on integrating

$$\int \frac{\sec^2 y}{\tan y} dy = + \int \frac{-e^x}{1-e^x} dx$$

$$\log \tan y = \log (1-e^x) + \log c$$

$$\cancel{\log} \tan y = \cancel{\log} c (1-e^x)$$

$$\tan y = c (1-e^x)$$

it is general sol of VSDE.

Q.No
7. Solve the DE

$$x\frac{dy}{dx} - y + x\sin\frac{y}{x} = 0 \qquad \div x$$

$$\frac{dy}{dx} - \frac{y}{x} + \sin\frac{y}{x} = 0$$

$$\frac{dy}{dx} = \frac{y}{x} - \sin\frac{y}{x}$$

it is Homogeneous Diff Eqn.

Putting $\frac{y}{x} = v \longrightarrow (1)$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \longrightarrow (2)$$

HDE: $\frac{dy}{dx} = \frac{y}{x} - \sin\frac{y}{x}$

$$\cancel{v} + x\frac{dv}{dx} = \cancel{v} - \sin v$$

$$x\,dv = -\sin v\,dx$$

$$\frac{1}{\sin v}\,dv = -\frac{1}{x}\,dx$$

on integrating

$$\int \operatorname{cosec} v\,dv = -\int \frac{1}{x}\,dx$$

$$\log(\operatorname{cosec} v - \cot v) = -\log x + \log c$$

$$\cancel{\log}\left(\operatorname{cosec}\frac{y}{x} - \cot\frac{y}{x}\right) = \cancel{\log}\,\frac{c}{x}$$

$$x\left(\operatorname{cosec} y/x - \cot y/x\right) = C$$

Q.No. 8. Solve the DE

$$(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}, \quad y = 0 \text{ when } x = 1$$

Sol: $(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$, $\div (1+x^2)$

$$\frac{dy}{dx} + \underbrace{\frac{2x}{1+x^2}}_{P} y = \underbrace{\frac{1}{(1+x^2)^2}}_{Q}$$

it is LDE (linear diff. eqn.)

$$\text{I.F. (integrating factor)} = e^{\int P dx}$$

$$= e^{\int \frac{2x}{1+x^2} dx} = e^{\log(x^2+1)} = x^2+1$$

Sol. of LDE

$$y \times IF = \int Q \times IF$$

$$y \times x = \int \frac{1}{(1+x^2)^{\cancel{2}}} \times \cancel{(1+x^2)} \cdot dx$$

$$yx = \int \frac{1}{1+x^2} dx$$

$$yx = \tan^{-1} x + c, \quad \text{general sol.}$$

Put $x = 1$, $y = 0$

$$0 = \tan^{-1}(1) + c$$

$$0 = \pi/4 + c \qquad \therefore c = -\pi/4$$

$\therefore$ Particular sol

$$yx = \tan^{-1} x - \pi/4$$

(9) if $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perp. such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ find $|\vec{a}+\vec{b}+\vec{c}|$?

Sol: $(\vec{a}+\vec{b}+\vec{c})^2 = \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\vec{a}\cdot\vec{b} + 2\vec{b}\cdot\vec{c} + 2\vec{c}\cdot\vec{a}$

$$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 0 + 0 + 0$$

$$\left\{\begin{array}{l} \text{if } \vec{a} \perp \vec{b}, \quad \vec{a}\cdot\vec{b} = 0 \\ \text{if } \vec{a}\cdot\vec{a} = |\vec{a}|\,|\vec{a}|\cos 0 = |\vec{a}|^2 \end{array}\right.$$

$$|\vec{a}+\vec{b}+\vec{c}|^2 = 3^2 + 4^2 + 5^2$$

$$|\vec{a}+\vec{b}+\vec{c}|^2 = 9 + 16 + 25$$

$$|\vec{a}+\vec{b}+\vec{c}|^2 = 50$$

$$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{50}$$

$$= 5\sqrt{2}$$

Q.No

(10.) The two adjacent sides of parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find unit vector parallel to its diagonal. Also find area. (10)

Sol: diagonal

$$\vec{d} = \vec{a} + \vec{b}$$

$$= (2\hat{i} - 4\hat{j} + 5\hat{k}) + (\hat{i} - 2\hat{j} - 3\hat{k})$$

$$= 3\hat{i} - 6\hat{j} + 2\hat{k}$$

unit vector

$$\hat{d} = \frac{\vec{d}}{|\vec{d}|}$$

$$= \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{9+36+4}} = \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{7}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix} = \hat{i}(12+10) - \hat{j}(-6-5) + \hat{k}(-4+4)$$

$$= 22\hat{i} + 11\hat{j} + 0\hat{k}$$

ar of $\|^{gram}$ $= |\vec{a} \times \vec{b}|$

$$= \sqrt{22^2 + 11^2}$$

$$= 11\sqrt{2^2 + 1^2}$$

$$= 11\sqrt{5} \text{ sq. units}$$

Q.No

11. Find the vector eqn of line passing through the point $(1,2,-4)$ and perp. to the lines (11)

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

Sol: The eqn of line through the point $(1,2,-4)$

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

$$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c} \longrightarrow (1)$$

it is perp. to given lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$

$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$3a - 16b + 7c = 0 \longrightarrow (2)$$

$$3a + 8b \pm 5c = 0 \longrightarrow (3)$$

on solving $\frac{a}{80-56} = \frac{-b}{-15-21} = \frac{c}{24+48}$

$$\frac{a}{\cancel{24}\,2} = \frac{\cancel{-}b}{\cancel{-36}\,3} = \frac{c}{\cancel{72}\,6}$$

$\therefore$ req. eqn of line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$

vector eqn of line $\vec{r} = (\hat{i}+2\hat{j}-4\hat{k}) + \lambda(2\hat{i}+3\hat{j}+6\hat{k})$

Q No - 12 Find the shortest distance bet lines

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$$

$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} \div (2s+1)\hat{k}$$

Sol: on simplification of line 1

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$$

$$= \hat{i} - t\hat{i} + t\hat{j} - 2\hat{j} + 3\hat{k} - 2t\hat{k}$$

$$= (\hat{i} - 2\hat{j} + 3\hat{k}) + t(-\hat{i} + \hat{j} - 2\hat{k})$$

and line 2

$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$$

$$= s\hat{i} + \hat{i} + 2s\hat{j} - \hat{j} - 2s\hat{k} - \hat{k}$$

$$= (\hat{i} - \hat{j} - \hat{k}) + s(\hat{i} + 2\hat{j} - 2\hat{k})$$

Point of lines

$$\vec{a}_1 = \hat{i} - $$

$$\vec{a}_2 = \hat{i} - \quad k$$

$$\vec{a}_2 - \vec{a}_1 = (\hat{i} - \hat{j} - \hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k})$$

$$= 0\hat{i} + \hat{j} - 4\hat{k}$$

direction of

$$\vec{b}_1 = -\hat{i} + j - 2\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$$= \hat{i}(-2+4) - \hat{j}(2+2) + \hat{k}(-2-1)$$

$$= 2\hat{i} - 4\hat{j} - 3\hat{k}$$

now, $\quad S.D. = \left| \dfrac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$

$$= \left| \frac{(0\hat{i} + \hat{j} - 4\hat{k}) \cdot (2\hat{i} - 4\hat{j} - 3\hat{k})}{\sqrt{4+16+9}} \right|$$

$$= \left| \frac{0 - 4 + 12}{\sqrt{29}} \right|$$

$$= \frac{8}{\sqrt{29}} \text{ units}$$

Q.No - 13 Find the eqn of plane through the intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ which is perp. to the plane $x-y+z=0$

Sol: The eqn of intersecting plane

$$P_1 + \lambda P_2 = 0$$

$$(x+y+z-1) + \lambda(2x+3y+4z-5) = 0$$

$$x+y+z-1+2\lambda x+3\lambda y+4\lambda z-5\lambda = 0$$

$$(1+2\lambda)x + (1+3\lambda)y + (1+4\lambda)z - (1+5\lambda) = 0 \quad \rightarrow (1)$$

given perp. plane $x-y+z=0$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$1\cdot(1+2\lambda) - 1(1+3\lambda) + 1(1+4\lambda) = 0$$

$$1+2\lambda \cancel{-1} - 3\lambda \cancel{+1} + 4\lambda = 0$$

$$3\lambda = -1$$

$$\lambda = -1/3$$

$\therefore$ req. eqn of plane

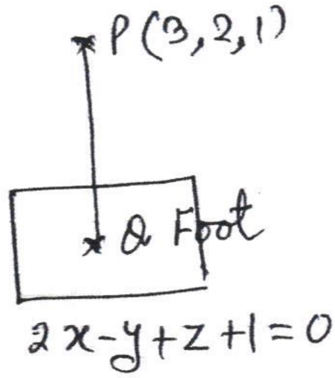
$$(1+2\lambda)x + (1+3\lambda)y + (1+4\lambda)z - (1+5\lambda) = 0$$

$$\left(1-\frac{2}{3}\right)x + \left(1-\frac{3}{3}\right)y + \left(1-\frac{4}{3}\right)z - \left(1-\frac{5}{3}\right) = 0$$

$$\frac{1}{3}x + 0 - \frac{1}{3}z + \frac{2}{3} = 0$$

Q No-(14) Find the coordinates of the foot of perp. and perp. distance of the point $P(3, 2, 1)$ from the plane $2x - y + z + 1 = 0$. Also find image.

Sol:



The eqn of PQ,

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1} = K \text{ (let)}$$

General Point on PQ,

$$\text{Foot, } Q = (2K+3, -K+2, K+1)$$

Put the point in plane

$$2x - y + z + 1 = 0$$

$$2(2K+3) - (-K+2) + (K+1) \pm 1 = 0$$

$$4K + 6 + K - \cancel{2} + K + \cancel{1} + \cancel{1} = 0$$

$$6K + 6 = 0$$

$$K = -1$$

$\therefore$ req. Coordinate of Foot $= (2K+3, -K+2, K+1)$

$$= (-2+3, 1+2, -1+1)$$

$$= (1, 3, 0)$$

Distance of PQ, $P(3, 2, 1)$ $Q(1, 3, 0)$

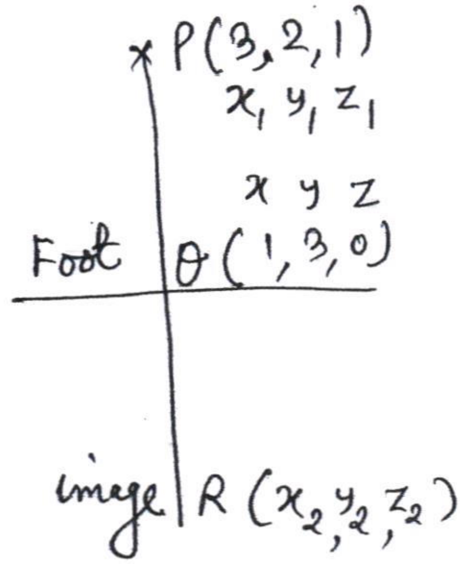
$$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_1-z_2)^2}$$

$$= \sqrt{(1-3)^2+(3-2)^2+(0-1)^2}$$

$$= \sqrt{4+1+1}$$

$$= \sqrt{6} \text{ units}$$

image of the point P, using mid point formula



$$x = \frac{x_1+x_2}{2}, \quad y = \frac{y_1+y_2}{2}, \quad z = \frac{z_1+z_2}{2}$$

$$1 = \frac{3+x_2}{2}, \quad 3 = \frac{2+y_2}{2}, \quad 0 = \frac{1+z_2}{2}$$

$$2 = 3+x_2, \quad 6 = 2+y_2, \quad 0 = 1+z_2$$

$$x_2 = -1, \quad y_2 = 4, \quad z_2 = -1$$

$\therefore$ image point $(-1, 4, -1)$.

Q-No-15 Find the area of the ellipse using integration.

$$9x^2+4y^2=36.$$

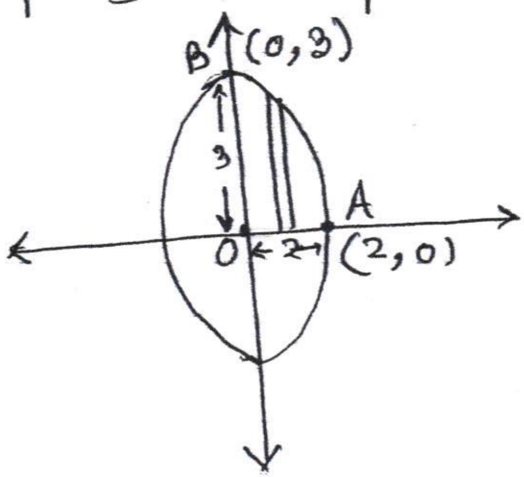
Sol: The eqn of ellipse

$$9x^2+4y^2=36 \qquad \div 36$$

$$\frac{9x^2}{36}+\frac{4y^2}{36}=\frac{36}{36}$$

$$\frac{x^2}{4}+\frac{y^2}{9}=1 \quad , \quad y=\frac{3}{2}\sqrt{4-x^2}$$

The graph of the ellipse



area using integration

$= 4 \times$ ar of OAB

$$=4\int_0^2 \frac{3}{2}\sqrt{4-x^2}\,.dx$$

$$=6\int_0^2 \sqrt{4-x^2}\,.dx$$

$$=6\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}.\sin^{-1}\frac{x}{2}\right]_0^2$$

$$=6\left[\frac{2}{2}\sqrt{4-4}+2\sin^{-1}\frac{2}{2}\right)-0\Big]$$

$= 6\left[0 + \cancel{2}\,\cancel{\sin^{-1}}\left(\cancel{\sin}\frac{\pi}{\cancel{2}}\right)\right]$

$= 6\pi$ sq. units

Q.No 16 Using integration, find the area of the region in the first quadrant enclosed by x-axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$;

Sol: The given eqns are

$$x^2 + y^2 = 4 \longrightarrow (1)$$

$$x = \sqrt{3}y \longrightarrow (2)$$

The point of intersection on solving eq (1) & (2)

$$x^2 + y^2 = 4$$

$$(\sqrt{3}y)^2 + y^2 = 4$$

$$4y^2 = 4$$

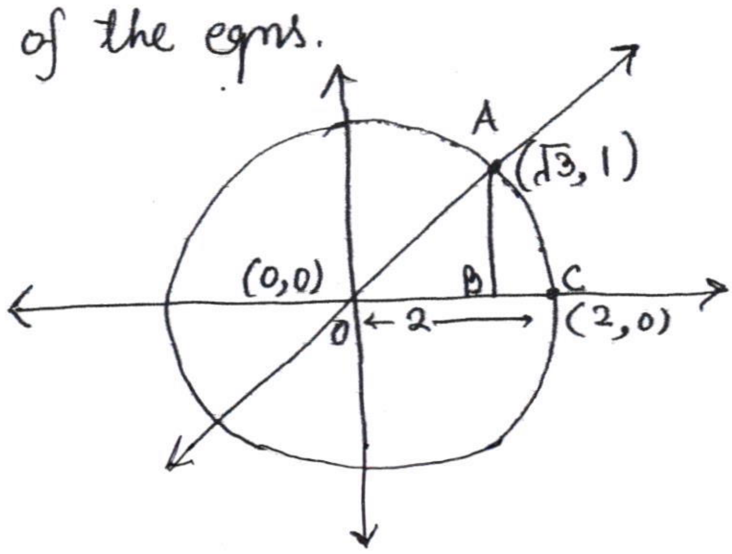
$$y = \pm 1$$

from eqn (2) $x = \sqrt{3}y$

$$x = \sqrt{3}(\pm 1) = \pm\sqrt{3}$$

$\therefore$ Point of intersection : $(\sqrt{3}, 1)$ and $(-\sqrt{3}, -1)$

Graph of the eqns.



area using integration

$= \text{ar. of line}_{(OAB)} + \text{ar. of circle}_{(ABC)}$

$$= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} \cdot dx + \int_{\sqrt{3}}^{2} \sqrt{4-x^2} \cdot dx$$

$$= \frac{1}{\sqrt{3}} \cdot \left[\frac{x^2}{2}\right]_0^{\sqrt{3}} + \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$$

$$= \frac{1}{\cancel{\sqrt{3}}}\left[\frac{(\sqrt{3})^{\cancel{2}}}{2} - 0\right] + \left[\left(\frac{2}{2}\sqrt{4-4} + 2\sin^{-1}\left(\frac{\cancel{2}}{\cancel{2}}\right)\right) - \left(\frac{\sqrt{3}}{2}\sqrt{4-3} + 2\sin^{-1}\frac{\sqrt{3}}{2}\right)\right]$$

$$= \cancel{\frac{\sqrt{3}}{2}} + 0 + 2\cancel{\sin^{-1}}\left(\cancel{\sin}\frac{\pi}{2}\right) - \cancel{\frac{\sqrt{3}}{2}} - 2\cancel{\sin^{-1}}\left(\cancel{\sin}\frac{\pi}{3}\right)$$

$$= \pi - \frac{2\pi}{3}$$

$$= \frac{\pi}{3} \text{ sq. units}$$

Q No-17 A manufacturer has three machine operators A, B, and C. The first operator A produces 1% of defective items where as other two operators produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A.

Sol: let the sample spaces

E_1 = Produced by machine A $\quad P(E_1) = \frac{50}{100} = 0.5$

E_2 = Produced by machine B $\quad P(E_2) = \frac{30}{100} = 0.3$

E_3 = Produced by machine C $\quad P(E_3) = \frac{20}{100} = 0.2$

Common event = A = defective item

$$P(A/E_1) = \frac{1}{100} = 0.01$$

$$P(A/E_2) = \frac{5}{100} = 0.05$$

$$P(A/E_3) = \frac{7}{100} = 0.07$$

using Baye's Theorem

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$

$$= \frac{0.5 \times 0.01}{0.5 \times 0.01 + 0.3 \times 0.05 + 0.2 \times 0.07}$$

$$= \frac{0.005}{0.034} = \frac{5}{34}$$

Q No-18 Two cards are drawn with out replacement from a pack of 52 cards. Find probability distribution of the number of Kings.

Sol: let the random variable = number of Kings in draw of 2 cards.

$$X = 0, 1, 2$$

Now $P(X=0) = P(\text{no King}) = P(\bar{K}\bar{K})$

$$= \frac{48}{52} \times \frac{47}{51}$$

$$= \frac{188}{221}$$

$P(X=1) = P(\text{one King and one non King})$

$$= P(K\bar{K}) + P(\bar{K}K)$$

$$= \left(\frac{4}{52} \times \frac{48}{51}\right) \times 2$$

$$= \frac{32}{221}$$

$P(X=2) = P(\text{Two Kings}) = P(KK)$

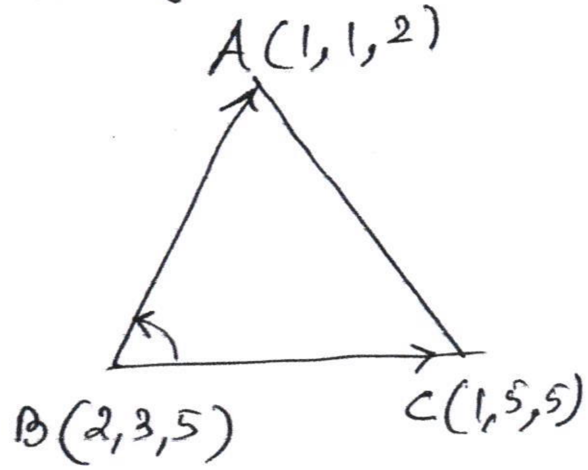
$$= \frac{4}{52} \times \frac{3}{51}$$

$$= \frac{1}{221}$$

$\therefore$ Probability Distribution

X	0	1	2
P(X)	188/221	32/221	1/221

Q No - 19 using vectors find the area of triang with vertices A(1, 1, 2) B(2, 3, 5) C(1, 5, 5)



Sol: $\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$

$= (\hat{i} + \hat{j} + 2\hat{k}) - (2\hat{i} + 3\hat{j} + 5\hat{k})$

$= -\hat{i} - 2\hat{j} - 3\hat{k}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= (\hat{i} + 5\hat{j} + 5\hat{k}) - (2\hat{i} + 3\hat{j} + 5\hat{k})$

$= -\hat{i} + 2\hat{j} + 0\hat{k}$

$$\overrightarrow{BA} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & -3 \\ -1 & 2 & 0 \end{vmatrix}$$

$= \hat{i}(0+6) - \hat{j}(0-3) + \hat{k}(-2-2)$

$= 6\hat{i} + 3\hat{j} - 4\hat{k}$

now, ar. of $\Delta ABC = \frac{1}{2} |\overrightarrow{BA} \times \overrightarrow{BC}|$

$= \frac{1}{2} \sqrt{36 + 9 + 16}$

$= \frac{1}{2} \cdot \sqrt{61}$ sq. units

Q No: - 20

Integrate: $\int \frac{x}{(x-1)^2 \cdot (x+2)} \cdot dx$

Partial Fraction:

$$\frac{x}{(x-1)^2 (x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$$

$$\frac{x}{\cancel{(x-1)^2 (x+2)}} = \frac{A(x-1)(x+2) + B(x+2) + C(x-1)^2}{\cancel{(x-1)^2 (x+2)}}$$

$$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2 \longrightarrow (1)$$

Putting $x = 1$, $\quad 1 = B(3) \quad \therefore B = 1/3$

$x = -2$, $\quad -2 = C(9) \quad \therefore C = -2/9$

$x = 2$, $\quad 2 = 4A + 4B + C$

$$2 = 4A + \frac{4}{3} - \frac{2}{9}$$

$$2 = 4A + \frac{12 - 2}{9}$$

$$2 = 4A + \frac{10}{9}$$

$$2 - \frac{10}{9} = 4A$$

$$\frac{8}{9} = 4A$$

$$\frac{2}{9} = A$$

now, integral becomes

$$\int \frac{x}{(x-1)^2(x+2)}\,dx = \int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \cdot dx$$

$$= \frac{2}{9}\int \frac{1}{x-1}\,dx + \frac{1}{3}\int \frac{1}{(x-1)^2}\,dx + \left(-\frac{2}{9}\right)\int \frac{1}{x+2}\,dx$$

$$= \frac{2}{9}\log(x-1) + \frac{1}{3}\cdot\frac{-1}{(x-1)} - \frac{2}{9}\log(x+2) + C$$

$$= \frac{2}{9}\log\left|\frac{x-1}{x+2}\right| - \frac{1}{3(x-1)} + C.$$

(C. S. Sharma)
PGT (Maths)
GHSS, Naharlagun
8729900980